

# **Math for Diverse Learners in the Elementary Classroom**

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## **Introduction**

I always laugh to myself when I see the anxious look on student teachers' faces when they first find out they have been assigned a split class for their practicum. I calmly try to explain to them that regardless of the number(s) beside their grade placement that each and every one of them has a split class...even if it is a straight grade class! It is unlikely that this puts their minds at ease...however, it is the reality.

The range of physical and mental abilities, personalities, attitudes, behaviours, cultural differences, gender differences, learning styles and day-to-day circumstances that may affect students' performance in today's classrooms pose significant challenges for teachers.

Is it possible to meet the needs of all of your students in mathematics? Is it possible for every student in your class to learn math? Do all students need to learn the same math? Are there more effective ways to teach math than others? What are some strategies for meeting that range of students in your math class? Does the same strategy that works for one student work for another? These are just some of the questions we need to ask about teaching math to diverse learners.

In this paper, I will not be addressing specific mathematical learning disabilities. Rather the discussion will focus on the general diversity found in our classrooms today in terms of teaching and learning the regular mathematics curriculum. This diversity includes, but is not limited to, intellectual, cultural and linguistic, gender, and enrichment issues. Practical adaptations and instructional strategies will be suggested for trying to address the range in your regular mathematics classroom, which are really best practices for the teaching of mathematics in general. One of the principles set out by the National Council of Teachers of Mathematics (NCTM, 2000) is the Equity Principle. All students must have the opportunity and adequate support to learn mathematics "regardless of personal characteristics, backgrounds, or physical challenges" (p.12). John Van de Walle, in his popular book, "Elementary and Middle School Mathematics: Teaching Developmentally" (Second Canadian Edition, 2008) says that "There are no methods of teaching mathematics for special children that differ fundamentally from the ways we teach all children." The reality is that best practices in mathematics are best practices period, regardless of the diversity of learners. Best practices in mathematics are those that teach for understanding. Teachers need to recognize that even within teaching for understanding, there will be different teaching and learning styles and different knowledge will be assimilated by each student. Teachers need to be aware that even though they "teach" the content, it may be "learned" in different ways.

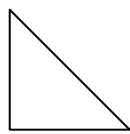
## Teacher Pedagogical Content Knowledge

In the mid-1980s, Lee Shulman and his colleagues introduced the notion of "pedagogical content knowledge" to refer to the special nature of the subject matter knowledge required for teaching (Shulman, 1986; Wilson, Shulman, & Richert, 1987). Teaching math not only requires that teachers have an understanding of the math concepts they are to teach, an understanding of some general learning theories, an understanding of the cognitive developmental levels of students at the grade level they teach, but also a sense of how best to teach this math to these students. Deborah Loewenberg Ball is an education professor and Dean of the School of Education at the University of Michigan. Her research focuses on mathematics instruction and on the nature of the mathematical knowledge needed for being an effective math teacher. She explains teacher pedagogical content knowledge like this, "The knowledge good math teachers need consists of more than knowing math well or understanding how children think at particular developmental stages. It comes from knowing how to apply mathematical knowledge, quickly, in ways that make sense to students." (Viadero, 2004, p.1). This knowledge includes knowing or learning what math topics students find interesting, or find easy or find challenging. It includes discerning what representation or what manipulatives might best be used to learn certain concepts. It includes a teacher's awareness of learners' typical errors and misconceptions, and then a timely teaching response.

The concept of pedagogical content knowledge, undeniably, plays a very important part in teaching math to diverse learners. How teachers adjust their teaching for certain math concepts or for certain students are variables that affect the learning process. Being aware of common student misconceptions, or more so specific misconceptions by individual students will enhance the learning process.

As an example, when young students are introduced to classification and patterns, simple attributes such as colour, shape and size are used. When older students are faced with more sophisticated classifications or patterns that involve intangible attributes such as function, use or relationship to something else, students may limit their thinking to tangible attributes that were first used to introduce patterns and may not recognize more abstract classifications or patterns. Another example may be that if patterns were consistently introduced or focused on as increasing patterns, some students may fail to recognize a pattern that is decreasing.

Orientation is another good example where a teacher's pedagogical content knowledge can help to interpret students' learning. For example, if students are introduced and limited to triangles presented only in this following standard orientation,

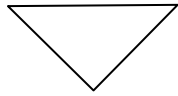


right triangle

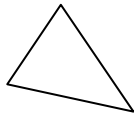


isosceles triangle

the teacher needs to be aware that students may not recognize the following as a right triangle and as an isosceles triangle, or at a younger age, as triangles at all.



right triangle



isosceles triangle

A teacher's pedagogical content knowledge will help her decide whether or not a student understands or is simply not interested or confused. Pedagogical content knowledge helps guide her choice of instructional strategies, which may be different from content area to content area, or from lesson to lesson, possibly from groups of students to groups of students and maybe even from individual student to individual student.

## **Instructional Approaches and Strategies**

### **Using Problems**

The best way to teach mathematics and attend to the range of students is to use the problem-based approach. Using problems with students' interests in mind, encourages ownership of the math involved as well as allows students to make connections, across the math strands and to the real world. A meaningful purpose has been set for the math i.e. "I need or want to solve this math problem because..." Research has shown that students who are more engaged with school and with mathematics are far more likely to be successful in school and in learning mathematics. (Nardi & Steward 2003).

Then students should be encouraged to use their own methods and personal strategies to solve problems. "Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies. In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought.... Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers." (Ministry of Education, 2007, p.19)

At some point, problem solving strategies such as making a list or drawing a diagram or working backwards should be modeled and even taught. However, the ultimate goal would be for students to assimilate these various strategies into their own personal repertoire and have them available to use at their discretion given the problem. That is, we should not teach that "These problems should be solved by drawing a picture, and these problems should be solved by working backwards." While teachers may introduce a specific problem solving strategy by using problems that are likely or more easily solved by using one strategy over another, the goal is to have students make the decision how to solve a problem. The focus on using a problem-based approach is on the process,

although it would be foolish to say that the focus is on the process with no regard to getting “a” correct answer. William Brownell, in 1946, said “A problem is not necessarily solved because the correct answer has been made. A problem is not truly solved unless the learner understands what he has done and knows why his actions were appropriate.” (in McIntosh & Jarrett, 2000), and that still holds true today. Diverse learners learn in different ways and solve problems in different ways. As a teacher of diverse learners, it is important to recognize that and even more important to ask students lots of questions about how they solved a problem and to listen carefully.

### **Asking Good Questions and Listening Carefully**

“Regardless of the classroom, it is always important to listen to your students.” (VandeWalle, 2008, p64). Try to find out what your students are thinking. Ask questions not only about what they are doing but why and how. No two students will have the same understanding of a concept or even a problem, because that understanding is based on the experiences each child has had.

You cannot effectively know what a child is thinking by looking at paper and pencil work unless you are asking questions alongside e.g. “Why did you start there? How did you know what to do next? What do these numbers mean? Could you have done this differently? Does your answer make sense?” By asking questions and listening carefully you will be in a better position to put into practice your teacher pedagogical content knowledge and deal effectively with individual students. Listening is an effective means of assessing “for” learning. Teachers need to ask more open-ended questions and ask students to explain their thinking and their answers. Students should also be encouraged to ask each other and themselves questions.

Australian educators, Sullivan and Lilburn (2002), focus on questioning in their book, “Good questions for math teaching: Why ask them and what to ask [K-6]”. The book is divided into two parts: Part One: The Importance of Questioning, and Part Two: Good Questions to Use in Math Lessons. Part One is divided into sections on a) What are Good Questions?, b) How to Create Good Questions, and c) Using Good Questions in Your Classroom. The good questions to use in math class that make up Part Two of this book are organized by math content area and age group. The authors suggest that, “our goals of education are for our students to think, to learn, to analyze, to criticize, and to be able to solve unfamiliar problems, and it follows that good questions should be part of the instructional repertoire of all teachers of mathematics.” It also follows that these should be the goals for every student in a math class of diverse learners. Teachers should ask questions to every student. The content can vary, the problems can vary, the difficulty can vary, the approaches can vary, but teachers can always ask “Why?”

### **Doing Math**

Students learn better by doing math rather than listening...and that applies to every student in the class. If students are engaged in a task, the learning becomes more meaningful.

Part of “doing math” includes the use of manipulatives. Manipulatives can help at various stages of learning: to help understand a problem or question, to help actually solve a problem, to reflect back to see if an answer is reasonable or unique, or to help students communicate and explain their thinking to others. Students should be encouraged to use manipulatives of their choice and should be able to use or not use them by choice. This last suggestion applies directly to diverse learners of math. Students learn math in different ways and manipulatives should be made available for all students. It should never be the case that students feel less smart if they have to ask to use manipulatives. An effective organizational method is to just have tubs of manipulatives somewhere in the classroom that are easily assessable for students to use when and if they want to.

Verbs such as explore, investigate, represent, construct, use, and explain all encourage students to “do” math. “When children are engaged in the kinds of activities suggested by (these verbs), it is virtually impossible for them to be passive observers. They will necessarily be actively thinking about the mathematical ideas that are involved.” (VandeWalle, 2008, p13). This type of involvement and action is attainable but may be different for every student. So, for example, while one student may be using base ten blocks, and another one graph paper, and another one a calculator, they could conceivably all be exploring the same place value concept. “Doing math” is important to learning...students just all don’t have to be doing the same “doing”.

### **Differentiating Tasks**

Unfortunately, a traditionally common practice in some math classrooms is to assign a drill and practice worksheet consisting of maybe 20 to 40 questions. Without delving into the appropriateness of this instructional practice (i.e. if it happens before understanding), consider the following. Let’s imagine the worksheet questions are good ones that further a student’s understanding of a math topic. Student A finishes the worksheet quickly. Student B has trouble with the first 10 questions. What does the hypothetical teacher do? Assign more of the same type of question to Student A because she finished early and can do more; and assign more of the same type of question to Student B until he “gets” them. This makes no sense. Even given the contrived worksheet idea, neither Student A nor Student B need more of the same. What Student A and Student B need are tasks that have been differentiated for their learning.

Sometimes it is easy to differentiate a task by varying the size of the numbers. This generally tends to work in one direction. If a student cannot complete a problem because he does not yet have a grasp of 3-digit numbers, perhaps attempting the same problem with 2-digit numbers will allow the student to focus on the process without getting stumped by the size of the numbers. However, increasing the size of the numbers for a student who already “gets” the problem has the potential to become busy work. A good place to start is to allow students the choice of size of number they wish to work with. For example, ask students to choose a number between 10 and 99 and then to multiply it by 3. Some students may be more comfortable choosing a smaller number or a multiple of ten or a number that they can easily add 3 times rather than multiply. Other students

may challenge themselves. Still others may choose numbers that are too easy for them in which case the teacher may have to suggest the differentiation.

Another way of differentiating a task is to present a number of related questions that aim to be at different levels of thinking. An instructional strategy involving questions that has been used more commonly in Reading is referred to as “On-the-line, Between-the-line, and Beyond-the-line.” “On-the-line” questions tend to be factual ones. Questions that are “Between-the-line” have students infer or figure out information provided by clues in the text and/or by making connections to their own personal knowledge and experiences. The “Beyond-the-line” questions challenge students to apply ideas from one situation to another. Can we use this instructional strategy in our math classes? Most certainly. This also ties in nicely with the previous section on questioning.

Example 1: Mandy bought a chocolate bar with \$1 and received 32¢ change.

On-the-line

How much did the chocolate bar cost?

Did you use addition or subtraction to solve this problem? Explain.

Between-the-line

What coins did Mandy receive in her change?

If Mandy had used the exact change to buy the chocolate bar, what coins might she have used?

Beyond-the-line

What are the fewest number and types of coins you could have so that you could pay the exact change for any chocolate bar priced between 50¢ and \$1?

Example 2: The length of a garden is 8 metres and the width is 3 metres.

On-the-line Questions:

What is the perimeter of this garden?

What is the area?

Between-the-line Questions:

What shape is this garden?

How can you determine the perimeter (area) in more than one way?

The perimeter of this garden is 22m. Can you make another garden with a perimeter of 22m?

Beyond-the-line Questions:

Is it possible to have different rectangles that have the same perimeter, but have different areas?

Is it possible to have different rectangles that have the same area, but have different perimeters?

Other ways that tasks can be differentiated is through the optional use of manipulatives, graph paper, calculators, partners etc. That is, allowing students to use something that will make the task easier for them. Same task, just with some accommodation.

Scaffolding is a way to make a task easier to understand, follow and/or solve. Templates or blackline masters can be given that show things such as the beginning of a solution or the steps to be followed or a graphic organizer that would be useful in solving the problem. Scaffolding is one practical suggestion for meeting the needs of diverse learners. There are many others.

### **Applying Practical Adaptations**

It is suggested in the British Columbia Integrated Resource Package for Mathematics K-7 that most of the prescribed learning outcomes are attainable by all students, including those with diagnosed special needs or second language learners. It is suggested that adaptations to strategies can be made to ensure success for those learners who struggle with the concepts (Ministry of Education, 2007). I believe it is a reasonable assumption to make that similar guidelines appear in other provincial curricula across Canada.

Numerous suggestions for adaptations and instructional strategies for diverse learners in math can be found in the literature. The following list of suggestions has been taken directly from a Masters Degree paper of a Graduate student of mine (Calado, 2007, 26-29). She researched and explored why students struggled in the acquisition of number skills and in her paper included a section on strategies to deal with the diversity of learners in a regular math class. Within this section, she cited a variety of sources as well as included strategies she herself and colleagues had used. The list is by no means conclusive, but is intended to provide ideas in supporting a student-centred learning environment.

#### Strategies and Adaptations For Reaching the Needs of Diverse Learners in Mathematics

- Teach concepts from the concrete to the abstract, using tangible materials when possible to represent vocabulary (Furner, Yaha, & Duffy, 2005). For example, when teaching addition, use found objects to represent numbers and model the process of adding them together, rather than simply “telling” students about the process.
- Relate mathematics to prior knowledge and background experience. This can be beneficial to English language learners when the math skill can be related to cultural experiences. Honouring students’ background knowledge and experiences will aid in boosting self-esteem because they have something to contribute to the lesson. This is also an important strategy for bridging the disparity between informal learning from home and formal education.
- Students who are struggling with a mathematical concept can benefit from teaching other students. Through explaining how they accomplish skills or

understand a concept, they are solidifying their knowledge and putting it into their own words, therefore committing it to their memory.

- Apply mathematics to daily life. For example, this is done through daily calendar exercises in primary classrooms, as well as through the use of menus and catalogues to teach money skills and multiplication. Applying mathematical skills to daily life also helps in the generalization of concepts.
- Have students draw their understanding of mathematical problems.
- Encourage “thinking aloud” of mathematical problems. This provides students with the opportunity to voice their thinking without being assessed. Thinking aloud allows students to verbalize concepts and perform metacognitive checking of procedures that strengthen problem-solving skills. This strategy also assists educators in pinpointing problem areas to revisit, as well as promotes self-correction (Furner, Yaha, & Duffy, 2005).
- Explain directions clearly and repeat key terms (Furner, Yaha, & Duffy, 2005).
- Make interdisciplinary connections with mathematics. This reinforces learning amongst subjects and generalizes skills.
- Model and demonstrate concepts using yourself and students as “actors”. For example lining students up in groups to demonstrate the concept of “less than” and “greater than”.
- Use literature to introduce mathematical concepts. Using picture books to introduce a skill aids in the generalization and connection of concepts. It also incorporates literacy skills and develops mathematical thinking in a non-traditional environment (Furner, Yaha, & Duffy, 2005).
- Keep into account, and address when possible, the multiple learning styles, such as auditory, visual, and kinesthetic.
- Provide opportunities for students to work at a pace that is comfortable and non-threatening. If a student needs enrichment and challenges, provide them with additional, or more complex problems. If a student needs assistance or remediation, provide them with simplified or fewer questions.
- Work with groups of students when possible to teach a skill that matches their level of learning. Provide opportunities for other students to reinforce their knowledge during this time through learning centres and games.
- Do research on your students. Review last year’s report cards, talk to teachers, and interview the students themselves. Find out what they know and where they



are coming from with their thinking. This will also provide an insight into potential areas of concern, or that will need additional emphasis.

### Providing Organization and Accommodations for All Students

At the start of this paper, I said that best practices for teaching mathematics generally are the best practices for teaching diverse learners in math. Again the literature abounds in suggestions for these best practices. I would highly recommend two articles in which the authors suggest very practical, very easy to implement ideas for mathematics accommodations for all students. The first article, in fact, is named just that, “Mathematics Accommodations for All Students” by Allison J. Fahsl, an assistant professor of special education at Southern Illinois University in Edwardsville. Fahsl describes how things like providing customized formats for graph paper can aid students with organizational problems, or how highlighting can help a student to discern operations in a multi-step problem.

For example, Fahsl suggests that the student can turn a standard lined piece of paper vertically to help assist with place value alignment (Figure 1) or highlight the placeholder (Figure 2). (Fahsl, 2007, p. 199, 200):

		2	3	6	
	+		4	1	

Figure 1

		3	5	8	
	+	1	2	6	

Figure 2

In another article, “Teach Mathematics: Strategies to Reach All Students” (Furner, Yahya, Duffy), the authors suggest 20 ways to help support the teaching of math to diverse learners, with a focus on the multimodal approach that incorporates multiple intelligences. Some of these ways were listed in the graduate paper cited above. Some other ways the authors have composed and gleaned from research and are:

- Relate math problems and vocabulary to prior knowledge and background.
- Encourage drawings to translate and visualize word problems.

- Encourage children to think aloud when solving word problems, and have students give oral explanations of their thinking, leading to solutions.
- Group students heterogeneously during cooperative learning.
- Make cultural connections for students when teaching mathematics.

The 20<sup>th</sup> way the authors suggest for reaching all students is really a summary of not only their 20 suggestions, but a summary of how all students can learn best in a classroom of diverse learners. Furner and his colleagues say that “Using auditory, visual, and kinesthetic teaching approaches for different learning styles enables teachers to reach more students than the traditional direct-instruction or paper and pencil drill and practice forms of instruction.” (p. 21)

## Summary

In reality, the idea of a diverse learner in math is really an anomaly. Each of us is a diverse learner in math. We all learn things differently from one another and we may learn math differently depending on the content strand or even from problem to problem. In order to address that diversity in a classroom, teachers need to become diverse teachers, recognizing what they know about child development, what they know about math, what they know about teaching math and determining what they can do to stir all that knowledge together to present a varied, rich environment in which the learning of each individual student is at the heart.

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